

Human Centricity and Norm Awareness in Cognitive Systems

Ph.D. Dissertation

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(Under the supervision of Professor Munindar P. Singh)

2 August, 2023

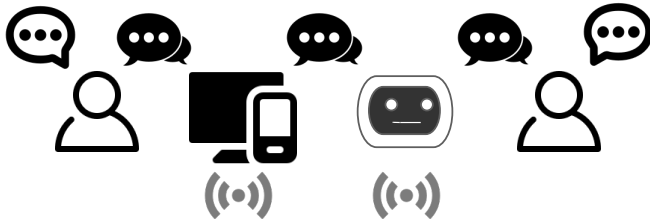
Department of Computer Science

NC STATE UNIVERSITY

Education

- Ph.D. in Computer Science, North Carolina State University, 2018
Fall to present
 - Norm Emergence in Systems of Cognitive Agents with Emotions and Values, proposed October 2021
 - Impact of Agent Interactions on Policy Selection in Social Dilemma Simulations, qualified September 2020
- M.Sc. in Computer Science, North Carolina State University, May 2018
- B.Sc. in Computer Engineering, National University of Tainan, Taiwan, June 2010

Introduction: Motivations



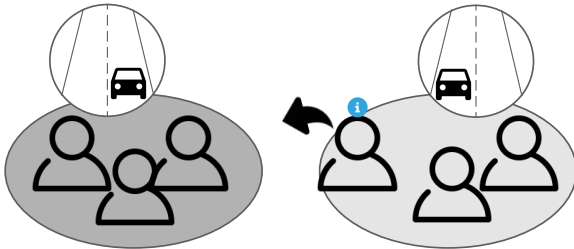
- Humans and agents form a multiagent system
- Norms regulate agent behaviors with sanctions
- What happens if humans are in the loop?
- What happens if human needs or the environment change over time?

Introduction: Challenges for Humans in Loop

- Human factors influence decisions and experience
 - The five human factors: **social**, **cognitive**, **emotional**, physical, and cultural
- Sanctions are often subtle, e.g., emotional expression or social exclusion
- Social signals have emerged in the form of verbal messages or subtle hints, transmitting normative information
- Values differ from person to person
- People need to comprehend and trust in AI output

Introduction: Challenges for Changing Environment

- Changing requirements or environment
 - More interconnection in MAS → Complexity of interactions increases drastically
 - Norms may change over time or over the environment



Introduction: Research Objective

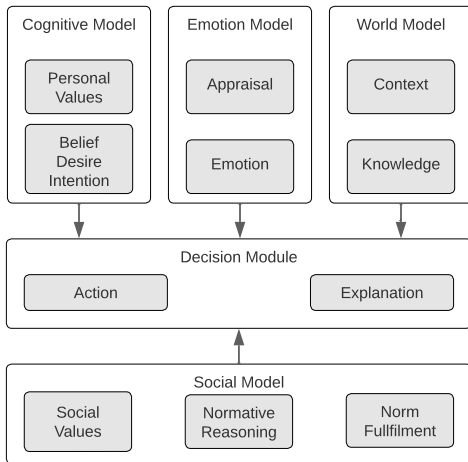
To accommodate humans in the loop and operate in dynamic environments

Thesis Statement

AI systems that consider human factors, such as **emotional expressions, social signals, social value orientation, and value-aligned decisions and rationales**, are more adept at accommodating humans in the loop, thereby enhancing the social experience

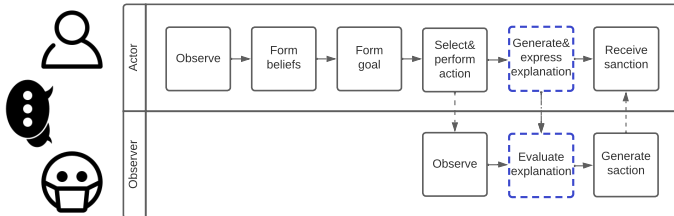
- Investigate emotional responses of agents to the outcomes of interactions [COINE @ AAMAS'21]
- Investigate messages and hints as drivers of subtle social learning [In prep for JAIR]
- Investigate the influences of social value orientation [COINE @ AAMAS'22]
- Investigate value-aligned decisions and rationales [In prep for JAAMAS]

Cognitive Framework



- Notions of social norms
 - Prescriptive norms describe how an individual should behave
 - Descriptive norms describe how most agents actually behave
 - Representation:
 - Norm(subject, object, antecedent, consequent) (Singh, 2013)
- Norm emergence: The majority of agents in society choose the same action
- Cooperation: Conforming to the existing norms or most agents' behaviors

General Interaction Among Agents In MAS



*Dashed rectangles apply when any explanation involves

*Solid rectangles indicate the processes of general interactions

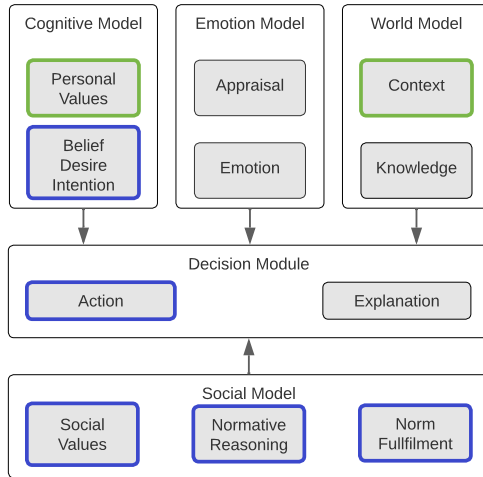
Fleur: Social Values Orientation for Robust Norm Emergence



Source: <https://twitter.com/springertoons/status/1281992099538165761>

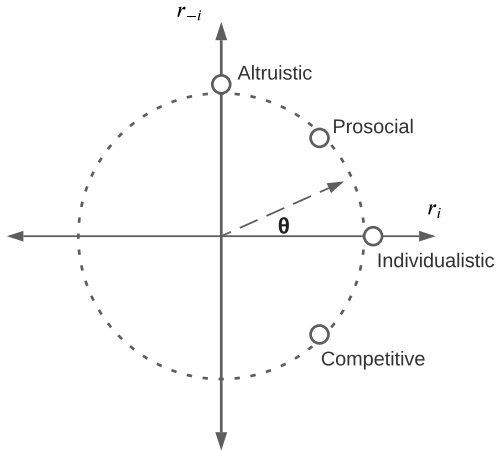
- Motivations
 - Interconnection in MAS indicates that one individual's behavior may affect another
 - Social Value Orientation (SVO): An individual's preference for resource allocation between self and others
 - Humans with different values evaluate the chosen actions subjectively and act to maximize their utility
- Objective: Incorporate individual preferences over self-interests and collective interests into decision-making
- RQ_{SVO} : How do social preferences, specifically social value orientation, influence norm compliance?

Fleur: Cognitive Framework



Fleur: SVO Ring with Reward Angle

Reward function of agent i : $reward_i = r_i \cdot \cos \theta + r_{-i} \cdot \sin \theta$



- Scenario: Agents interact with one another and decide whether to wear a mask based on preference, health state, and SVO
- Simulate with varying agent societies: Altruistic, Prosocial, Proself, Competitive, and Mixed society
- Characteristics of agent society
 - Prescriptive norm: Mask-wearing mandate
 - Different distribution of social value orientation among agents
 - Agents' health states and the chosen action determine the payoff

- Compliance
 - $M_{\text{Compliance}}$: The percentage of agents who satisfy the existing norm
- Social Experience
 - $M_{\text{Social Experience}}$: The total payoff of the agents in a society
- Invalidation
 - $M_{\text{Invalidation}}$: The percentage of agents who do not meet their preferences in a society

- $H_{\text{Compliance}}$: Social value orientation positively affects norm compliance with prosocial norms
- $H_{\text{Social Experience}}$: The distribution of social value orientation positively affects social experiences in a society
- $H_{\text{Invalidation}}$: Social value orientation negatively affects the tendency to meet personal preference and social experiences

Tests for Statistical Significance

- Independent t-test
- Glass's Δ

Result: Prosocial and Altruistic agents societies have higher compliance. A competitive infected agent may choose not to wear a mask when interacting with other healthy agents, leading to lower compliance in the mixed society

Compliance: % of agents who satisfy the existing norm

	S_{mixed}	$S_{altruistic}$	$S_{prosocial}$	$S_{selfish}$	$S_{competitive}$
Compliance	63.40%	69.70%	70.25%	65.10%	54.08%

Result: The mixed society has similar results as the selfish society. Whereas 50% of the mixed-agent society agents are altruistic and prosocial agents, the 25% of competitive agents would choose to minimize others' payoff without hurting their self-interests

Social Experience: The total payoff of the agents in a society

	S_{mixed}	$S_{altruistic}$	$S_{prosocial}$	$S_{selfish}$	$S_{competitive}$
Social Experience	0.448	0.554	0.566	0.470	0.221

Result: The selfish and competitive agents in the mixed society decreased the invalidation

Invalidation: % of agents who do not meet their preferences in a society

	S_{mixed}	$S_{altruistic}$	$S_{prosocial}$	$S_{selfish}$	$S_{competitive}$
Invalidation	0.296	0.334	0.323	0.269	0.289

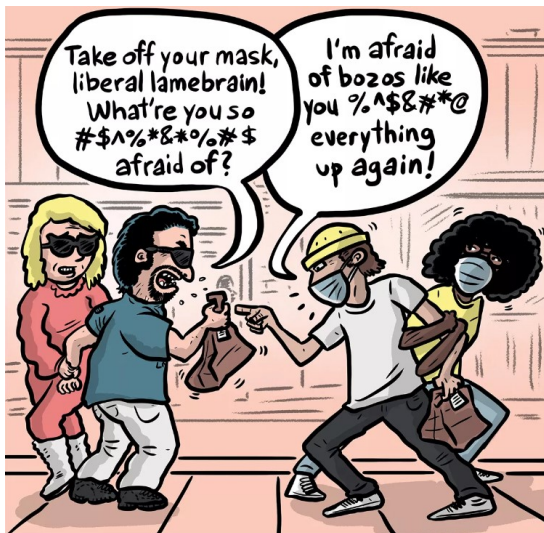
Study Summary

Incorporating Social Value Orientation enables better social experience and robust norm emergence

- Aligning with social preferences enables AI to make ethical decisions and be responsible for human needs
- Altruistic and prosocial agents adhere to the prosocial norm and enjoy more positive social experiences at the cost of themselves
- Policy makers may define appropriate sanctions to motivate the competitive and selfish agents to follow the norms

Exanna: Decision and Rationale with Values

Exanna: Scenario

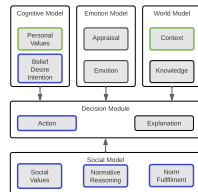
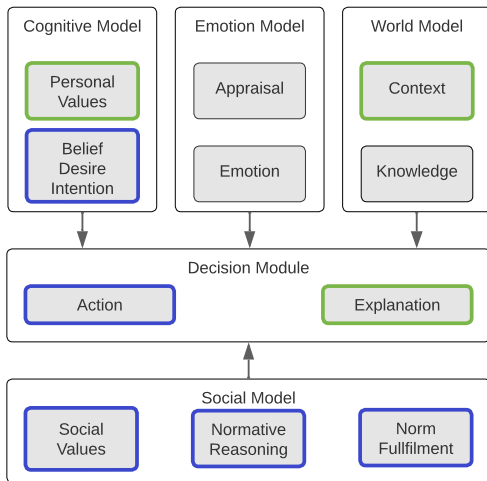


Source: <https://www.latimes.com/opinion/story/2022-08-07/mask-wearing-public-anger-comic>

- Motivations
 - Agents provides rationales for their decisions would be interpretable and reliable
 - Justifying behavior via revealing information can resolve social conflict and enhance individual gain
 - Verbose rationales may be diverging and not convincing, leading to information overload
 - Invaded or uncomfortable feelings for privacy breaches
 - Rationales or information aligned with values best justify one's behaviors

- Objective: Incorporating values into behavior justification
- RQs
 - RQ_{Goal Adherence}: Do value-aligned rationales increase adherence to the original goal?
 - RQ_{Conflict Resolution}: Do value-aligned rationales increase the social resolution?
 - RQ_{Privacy Loss}: Does value-aligned rationales reduce privacy loss?

Exanna: Cognitive Framework



Compare to Fleur

- Context:
 - The information that characterizes the situation of an entity
 - Include observable and nonobservable attributes (keep private from others)
 - Some attributes associated with values
- Decision rule: The mapping between an observation of context and a reasonable action, represented as if-then logic
 - Format: if *antecedent* then *consequence*
 $\{ \text{InfectionRisk}=\text{No risk} , \text{InteractWith}=\text{Colleague} \} \Rightarrow \text{Not Wear}$
- Rule Learning: Evolving rules from interactions or dataset
- Value preference
 - A preference order over different values for one context
 - Numbers in one value preference add up to 1

Exanna: Method (1)

- Decision making
 - Aggregated payoff with all corresponding values

$$f = \sum_i^{values} v_i \times r_{R \times C_y} \quad (1)$$

Agent 1:Agent 2	C1	C2
R1	r_{R1C1}	r_{R1C2}
R2	r_{R2C1}	r_{R2C2}

- Rationale Generation
 - Evolve and learn decision rules as the base rationale with XCS, a learning algorithm combines reinforcement learning and genetic algorithm
 - Rule discover: Crossover and mutation creates more general or more specific rules by randomly adding or removing factors in antecedent
 - Subsume rules: Replace with a more general rule that has less prediction error
 - Action selection: Select the action with best-aggregated fitness
 - Reveal necessary information in rationales
 - Remove private factors that are not related to presented values from the aggregated rules

- Rationale Evaluation
 - Update beliefs based on the received rationale
 - Make an analogous decision based on beliefs
 - Accept the rationale if the decision matches the observed action
 - Otherwise, reject the rationale
 - Acceptance and rejection of rationales lead to sanctions

- Scenario: Agents move randomly and decide whether to wear a mask based on personal preference, health states, and value preference
- Simulate with varying agent societies: Share All, Share Decision Rules, and Share Value-Aligned Rules society
- Characteristics of agent society
 - Different strategies to explain agents' behaviors
 - Evaluate observed behaviors referring to received rationales
 - Agents form goals based on values
 - 50% of agents value health and 50% of agents value freedom in each society

Measures:

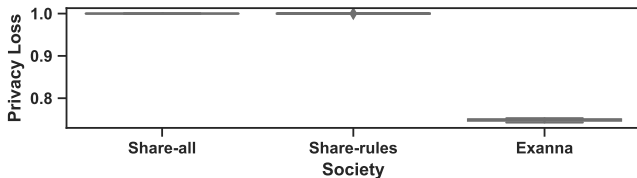
- Goal Adherence
 - $M_{\text{Goal Adherence}}$: The degree of adherence to each agent's goal
- Conflict Resolution
 - $M_{\text{Conflict Resolution}}$: The percentage of conflict resolution in society
- Social Experience
 - $M_{\text{Social Experience}}$: The aggregation of payoff an agent receives for its behavior
- Privacy Loss
 - $M_{\text{Privacy Loss}}$: The proportion of hidden information shared during an interaction

- $H_{\text{Goal Adherence}}$: Exanna provides higher goal adherence than baseline societies
- $H_{\text{Conflict Resolution}}$: Exanna provides higher conflict resolution than baseline societies
- $H_{\text{Social Experience}}$: Exanna provides better social experience than baseline societies
- $H_{\text{Privacy Loss}}$: Exanna takes lower privacy loss compared to baseline societies

Tests for Statistical Significance

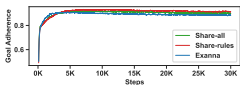
- Independent t-test
- Glass's Δ

Results: Exanna Yields Less Privacy Loss When Providing Rationale

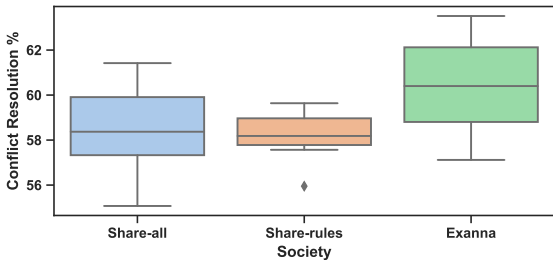


Results: Exanna Has Higher Conflict Resolution in Cases Where Agents Deviate from their Goals

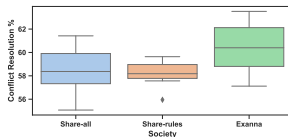
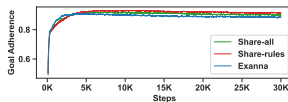
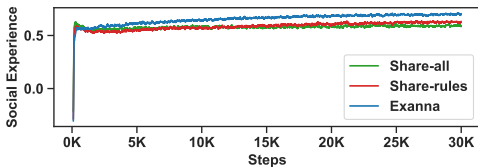
Exanna trades goal adherence for conflict resolution



Goal Adherence

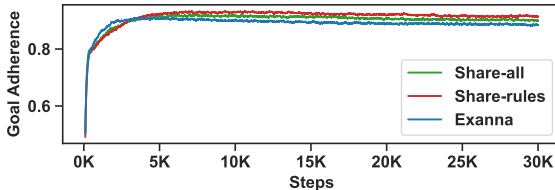


Results: Exanna Yields Better Social Experience and Conflict Resolution at the Expense of Goal Adherence



Results: Exanna Yields Lower Goal Adherence than Other Societies

- Exanna generates lower goal adherence than Share-Decision Rules society For less convincingness from being conservative
- Share All society has lower goal adherence than Share-Decision Rules society for distraction from information overload



Study Summary

Providing rationales with the concerns of value preferences leads to (1) deviation from goals, (2) higher conflict resolution, (3) less private loss, and (4) better social experience

- Value-aligned explanations ensure the AI system's decisions are consistent with human values
 - Highlight what an agent cares
 - No unnecessary sacrifice of private attributes

- Considering human factors leads to higher social experience in terms of a single agent
- Regarding MAS, Considering human factors promotes cooperation

- Going deeper into understanding the causal connections that exist between decisions and human factors
- Investigate how different costs of information influence decisions for more precise and reliable action suggestions and rationale construction
- Information suppression may be acceptable in some cases
- Having agents decide what to share and when to share increases strategies' flexibility
- Investigate the relationship between social norms and different social signals

Thank You

This research is based upon work supported by the National Science Foundation under Grants No. IIS-1908374 and No. IIS-2116751

General Reproducibility Details

Hardware:

- 32 GB RAM
- GPU NVIDIA GTX 1070 Ti

Framework:

- MASON (Java)
- Mesa (Python)

Noe: Enforcing Social Norms with Expressed Emotions

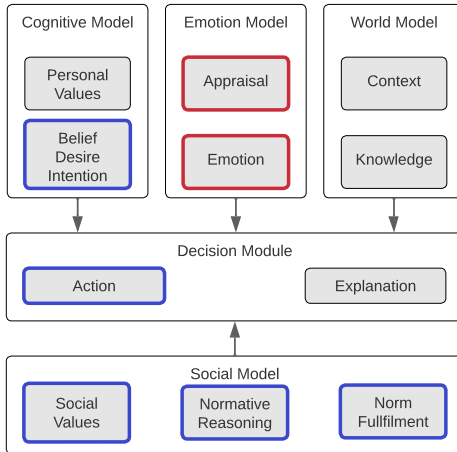
Noe: Introduction and Motivation



Source: <https://www.shutterstock.com/image-illustration/3d-smart-red-man-jumps-queue-134805779>

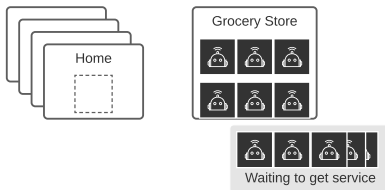
- Motivation:
 - Sanctions in real world are usually subtle
 - General thinking and problem-solving incorporate the influence of emotions (Simon, 1967)
- Objective: Incorporate expressed emotions in decision-making
- RQ_{emotion} : How does modeling the emotional responses of agents to the outcomes of interactions affect the emergence of norms and social welfare?

Noe: Cognitive Framework



Noe: Evaluation with Line-Up Simulations

- Simulate with varying agent societies: Obedient, Anarchy, Sanctioning and Noe society
- Appraisal: Based on norm satisfaction or violation
- Characteristics of agent society
 - Prescriptive norm: Line up to get service
 - Sanctions and **expressed emotions** that emerge from the evaluation of chosen actions
 - Expressed emotions serve as intrinsic reward (self-directed emotion) and extrinsic reward (other-directed emotion)



Measures:

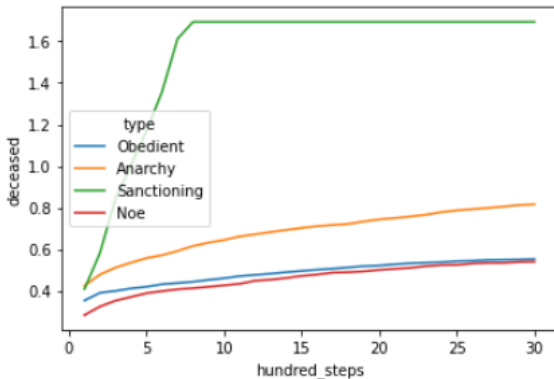
- Social welfare
 - M_{Deceased} : Cumulative number of agents deceased
 - M_{Health} : Average health of the agents
- Norm satisfaction
 - M_{Cohesion} : Proportion of norm instances that are satisfied
- Social experience
 - $M_{\text{Waiting time}}$: Average waiting time of agents in the queues

Noe: Simulation Results

		Obedient	Anarchy	Sanctioning	Noe
M_{Deceased}	\bar{X}	55.30	81.60	169.30	54.00
	p-value	<0.01	<0.01	<0.01	–
	Δ	0.65	3.10	15.53	–
M_{Health}	\bar{X}	79.27	79.50	86.26	79.00
	p-value	0.52	0.46	8.45	–
	Δ	0.18	0.21	3.34	–
M_{Cohesion}	\bar{X}	–	0.22	0.88	0.99
	p-value	–	<0.01	<0.01	–
	Δ	–	102.43	13.67	–
$M_{\text{Waiting Time}}$	\bar{X}	8.95	5.45	2.55	8.95
	p-value	0.98	<0.01	<0.01	–
	Δ	0.01	40.82	76.68	–

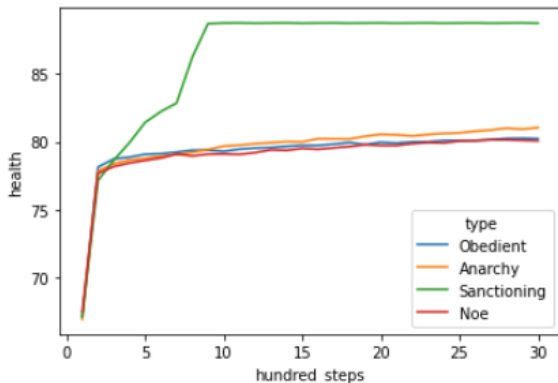
Results: Fewer Agents Die in Noe than in Other Societies

Metric: Cumulative number of agents deceased



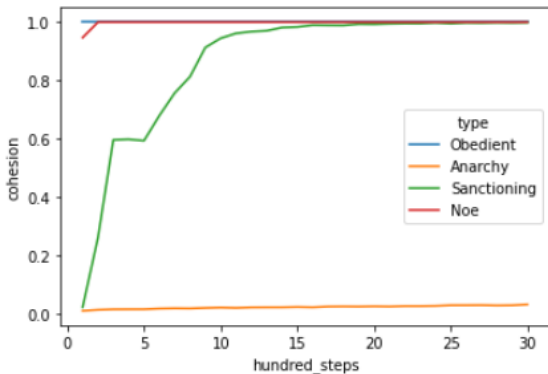
Results: Sanctioning Society Yields Higher Health State ... But at the Expense of More Deaths

Metric: Average health state of the agents



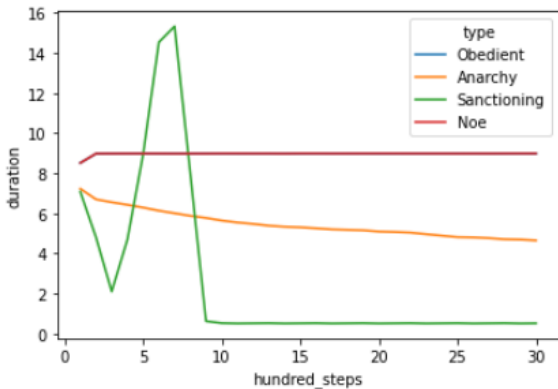
Results: Noe Yields Higher Cohesion than Other Agent Societies

Metric: Cohesion (Proportion of norm instances that are satisfied)



Results: Noe Has Similar Waiting Time as Obedient Society

Metric: Average waiting time of agents in the queues



Study Summary

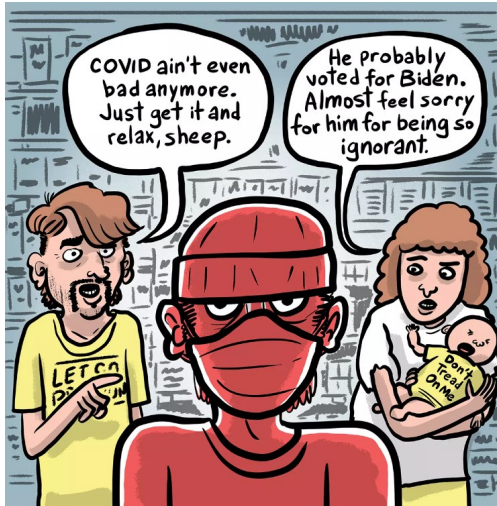
Agents who incorporate expressed emotions are more willing to comply with norms than those who do not

- Expressed emotions act as a positive or negative reinforcement mechanism for specific behaviors
- Noe enables the incorporation of expressed emotions as sanctions in decision-making

Ness: Normative Information from Tell and Hint

Ness: Scenario

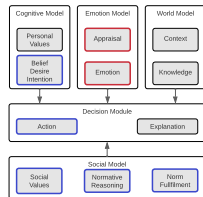
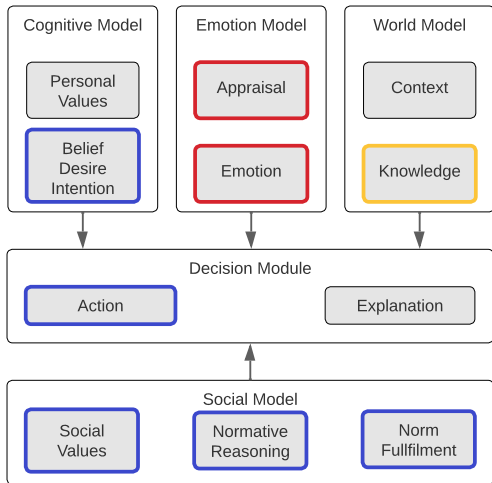
When there's a norm of not wearing a mask . . .



Ness: Introduction and Motivation

- Motivations
 - Social signals are reactions to norm satisfaction or norm violation
 - Social signals provide natural drivers for norm emergence
 - Normative information conveyed through a social signal promotes cooperation in MAS
 - Social signals can be realized in three main ways: *sanction*, *tell*, and *hint*
- Objective: Incorporate normative information from social signals into decision-making
- RQ_{information}: How does considering soft signals such as hints and tell in addition to sanctions influence norm emergence?

Ness: Cognitive Framework



Compare with Noe

Ness: Key Concepts

- Reward Shaping (Ng et al., 1999) provides additional “shaping” reward from deterministic reward function

$$r'_{final} = r + F$$

where r is the standard reward function in reinforcement learning and F is the shaping reward function

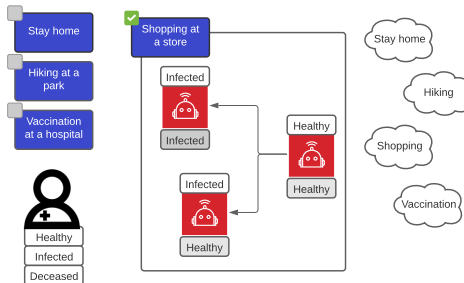
- With messages or hints, F defines the difference of potential values

$$F(s, a, s', a') = \gamma \Phi(s', a')\kappa - \Phi(s, a)$$

where Φ is a potential function that gives hints on states. κ defines the certainty of potential reward from the knowledge or information

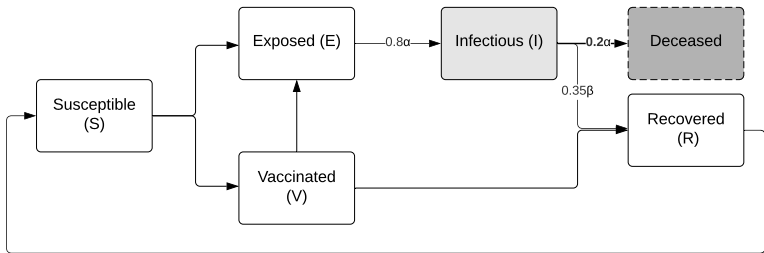
Ness: Evaluation with COVID Simulations

- Simulate with varying agent societies: Primitive, Sanction, Hint, Tell, and Ness society
- Characteristics of agent society
 - A combination of three kinds of social signals
 - Prescriptive norm: Stay self-quarantine if infected
 - Sanction (Material punishment): Send to forced quarantine at a low probability



Ness: Disease Model

Susceptible-Exposed-Infected-Recovered-Vaccinated (SEIRV)
model (Yang and Wang, 2020; Annas et al., 2020)



- α controls the probability to be infected based on vaccination
- β controls the recovering rate based on agent activity
- Healthy agents, cover susceptible, exposed and recovered, are not infected
- Infectious includes three subclasses: Asymptomatic, mildly symptomatic, and critical symptomatic

Ness: Agent Societies

Society	Sanctioning	Shaping Reward	Emotion
Baseline 1. PRIMITIVE	✗	✗	✗
Baseline 2. SANCTION	✓	✗	✗
Baseline 3. TELL	✓	✓	✗
Baseline 4. HINT	✓	✗	✓
Ness	✓	✓	✓

Shaping Rewards come from normative information

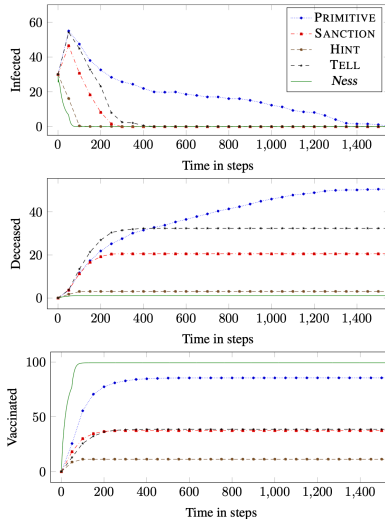
Ness: Information Balance

Societies: Signals	Sanction	Tell	Hint	Hint w/ shaping reward
PRIMITIVE	0%	0%	0%	0%
SANCTION	38%	0%	0%	0%
TELL	20%	36%	0%	0%
HINT	20%	0%	12%	0%
Ness	20%	0%	0%	10%

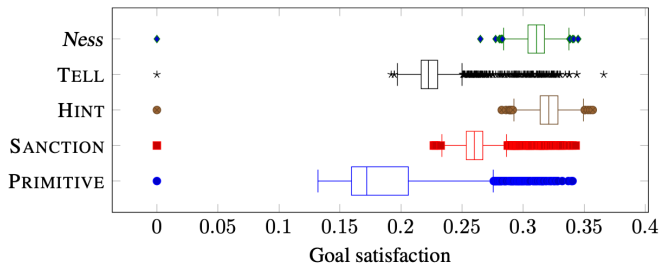
- More learning channels improve learning efficiency
- We balance the information an agent can access by adjusting the expected payoff to achieve comparability

- Disease control
 - M_{Healthy} : The percentage of agents who are healthy
 - M_{Infected} : The percentage of agents who are infected
 - M_{Deceased} : The percentage of who are deceased
 - $M_{\text{Total infections}}$: Total number of infections in societies
 - $M_{\text{Vaccinated}}$: Percentage of vaccinated agents
- Goal
 - M_{Goal} : The average goal satisfaction among agents
- Isolation
 - $M_{\text{Isolation}}$: The percentage of self-isolation among infected agents
 - $M_{\text{Forced quarantine}}$:
 - Number of agents who are forced to quarantine at home
 - This measure maps to the sanction signal type

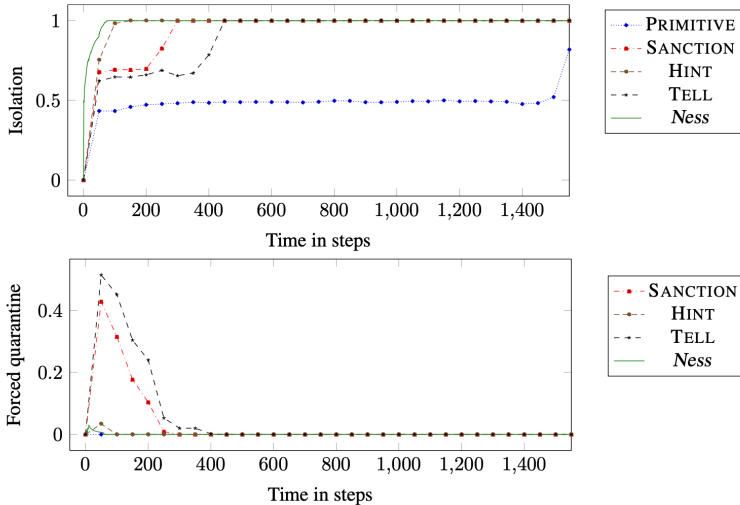
Results: Ness Yields Better Disease Control and Higher Vaccination Rate



Results: Ness Yields Better Goal Satisfaction Than Tell, Sanction, and Primitive Societies



Results: Ness and Noe Yield Higher Norm-Compliance and Lower Forced Quarantine than Other Societies



Ness: Detailed Results (1)

		PRIMITIVE	SANCTION	HINT	TELL	Ness
H _{Disease control}	M _{Infected}	13.281	2.634	0.411	4.205	0.157
	Δ	-0.973	-0.271	0.085	-0.330	-
	M _{Healthy}	46.294	77.602	96.622	65.082	98.750
	Δ	18.259	3.414	0.776	4.784	-
	M _{Deceased}	41.034	19.764	2.967	30.713	1.093
	Δ	-3.346	-6.123	-7.450	-5.316	-
	M _{Infections}	48.335	13.840	2.221	20.474	0.891
	Δ	-2.664	-6.925	-10.730	-5.842	-
	M _{Vaccinated}	82.452	36.743	11.185	37.430	98.734
	Δ	1.518	18.181	143.254	13.261	-

Ness: Detailed Results (2)

		PRIMITIVE	SANCTION	HINT	TELL	Ness
$H_{\text{Isolation}}$	$M_{\text{Isolation}}$	0.610	0.965	0.993	0.934	0.998
	Δ	1.777	0.326	0.101	0.450	—
	$M_{\text{Forced quarantine}}$	—	0.026	$8.5e - 04$	0.040	$1.75e - 04$
	p-value	—	<0.001	< 0.01	<0.001	—
	Δ	—	-0.268	-0.075	-0.313	—
H_{Goal}	M_{Goal}	0.187	0.262	0.321	0.227	0.311
	Δ	3.128	3.445	-1.012	3.929	—

Study Summary

Ness agents effectively avoid undesirable results and yield higher satisfaction than baseline agents despite requiring an equivalent amount of information

- Normative information from soft signals like hints and messages helps to regulate behaviors
- Incorporating normative information from social signals supports norm emergence

Ness: Hyperparameters

Parameter	Value	Comment
Learning rate α	0.001	
Discount factor γ	0.900	
Simulation step per action	1.000	
Infection %	0.300	The default fraction of infected agents in a society
Certainty of potential reward	0.300	value for κ for certainty of possible sanctions from normative information through hints
Certainty of potential reward	0.500	value for κ for certainty of possible sanctions from normative information messages

Fleur: Detailed Results (1)

		Compliance	Social Experience	Invalidation
S_{mixed}	\bar{X}	63.40%	0.448	0.296
	p-value	–	–	–
	Δ	–	–	–
$S_{altruistic}$	\bar{X}	69.70%	0.554	0.334
	p-value	< 0.001	< 0.001	< 0.001
	Δ	0.660	0.612	0.464
$S_{prosocial}$	\bar{X}	70.25%	0.566	0.323
	p-value	< 0.001	< 0.001	< 0.05
	Δ	0.718	0.677	0.326

Fleur: Detailed Results (2)

		Compliance	Social Experience	Invalidation
$S_{selfish}$	\bar{X}	65.10%	0.469	0.269
	p-value	0.218	0.424	< 0.05
	Δ	0.178	0.122	0.329
$S_{competitive}$	\bar{X}	54.08%	0.221	0.289
	p-value	< 0.001	< 0.001	0.541
	Δ	0.977	1.313	0.088

Fleur: Hyperparameters

Parameter	Value
Population size	40
Simulation step per action	1
Training steps	500,000
Evaluation steps	100
Learning rate α	0.001
Discount factor γ	0.9

Exanna: Processes of XCS

- Matching: A process that matches the current context and all rules/classifiers to generate a match set
- Covering: A process that guarantees diversity via adding a random classifier whose conditions match the current context
- Action selection: This process returns the action with the highest fitness-weighted aggregation of reward if in exploitation mode
- Formation of action set: The action set includes all classifiers that propose the chosen action based on the match set
- Updating classifier parameters: An agent updates the rule parameters (e.g., accuracy and fitness) based on the received payoff
- Subsumption: A process that replaces offspring rules with more general parent rules if it exists and with a minor prediction error
- Deletion: Each action set has the same maximum number of rules and XCS removes the low-fitness rules

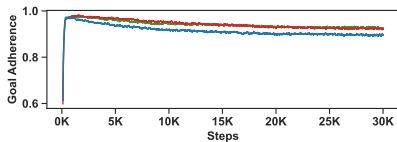
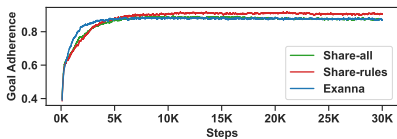
Exanna: Hyperparameters

Parameter	Value
Population size	200
Learning rate	0.1
Don't care probability	0.3
Accuracy threshold	0.01
Fitness exponent	5
Genetic algorithm threshold	25
Mutation probability	0.4
Crossover probability	0.8
Experience threshold for deletion	20
Experience threshold for subsumption	20
Fitness falloff	0.1

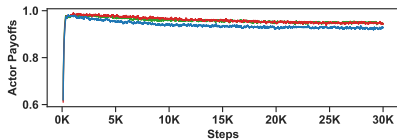
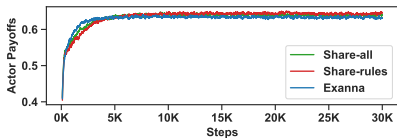
Exanna: Detailed Results

		Share All	Share Decision Rules	Exanna
$M_{\text{Goal Adherence}}$	\bar{X}	0.901	0.914	0.885
	p-value	0.083	< 0.01	–
	Δ	–1.594	–2.891	–
$M_{\text{Conflict resolution}}$	\bar{X}	0.585	0.582	0.604
	p-value	< 0.001	< 0.001	–
	Δ	1.803	3.106	–
$M_{\text{Social Experience}}$	\bar{X}	0.591	0.624	0.699
	p-value	< 0.001	< 0.001	–
	Δ	1.803	3.106	–
$M_{\text{Privacy Loss}}$	\bar{X}	1.000	0.999	0.749
	p-value	< 0.001	< 0.001	–
	Δ	∞	–11,896.523	–

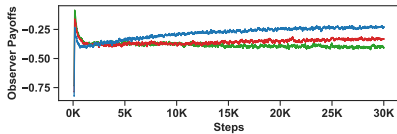
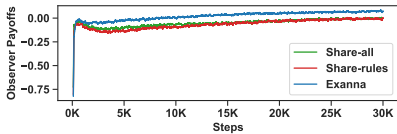
Exanna: Goal Adherence by Agent Types



Exanna: Payoff of Actors by Agent Types



Exanna: Payoff of Observers by Agent Types



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